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AN APPLICATION OF DYNAMIC MODELING TO THE SEA SHORE ROTATION PLANNING PROBLEM IN THE NAVY†

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Abstract—The Navy sea shore rotation planning problem is presented as multicriteria dynamic network optimization problem with side constraints. Using the simplex primal optimization code of Professor J. Kennington and a specially designed heuristic rounding routine we developed a computer model to run practical problems with different input parameters.

INTRODUCTION

The United States Navy is considering a major change in policy in rotating its enlisted personnel between sea duty and shore duty. Instead of the traditional fixed lengths of assignments (tour lengths) at sea and shore, the Navy wants to test more flexible policies to better align a dynamic personnel inventory with the dynamic manpower requirements of an evolving operating force. Enlisted rotation managers must respond to both long-range policy goals and near-term fluctuations in personnel vacancies. This paper develops an improved methodology which allows tour lengths to be flexible and dynamic to meet the Navy's personnel readiness needs.

Previous attempts to model the dynamics of Navy sea/shore rotation have not been entirely successful. Sorensen and Jones [1] approached the problem through statistical simulation, but this approach was not able to model the dynamic property of the problem nor was it capable of introducing multiple objectives and exploring policy tradeoffs. Charnes and Cooper [2] approached the problem as a generalized network with gain and losses. They use a goal programming formulation for objective function and try to minimize deviations from annual manning goals. Their model was developed with a "fixed tour" frame of reference.

APPROACH

Our approach is to model the Navy sea/shore rotation problem over time as a dynamic economic system. For the mathematical setting of the problem, we use the von Neumann approach for modeling of a dynamic discrete optimization system [3]. For meeting the multiple criteria character of the problem, we use a preemptive-like weight approach with the opportunity to change the order of objectives for different runs of the model [4]. For model implementation on the computer, we present the model as a network with side constraints and using the Kennington code for optimization [5] with a subsequent rounding routine specially designed by us for this kind of optimization. For the purpose of verification of the model, we use statistical data for the Aviation Antisubmarine Warfare Technician (AX) occupation (rating).

†The viewpoint and conclusion expressed in this paper are those of the writers and are not to be considered as official or as reflecting those of the Navy Department or the Naval Services.

MODEL FORMULATION

In the von Neumann modeling of a dynamic system, time is assumed to be discrete $t = 1, 2, \ldots, T$ where T is the planning horizon. The state of the model is a vector x in the positive orthant of n-dimensional Euclidian space R^n_+. The trajectory in the closed model is a sequence $\{x(t)\}_{t=t}^{t=T}$ such that $(x(t), x(t+1)) \in Z$ where Z is the given set of feasible pairs (technological set). The set Z is a subset in $R^n_+ * R^n_+$. The feasible set Z is described by a finite number of elementary processes $(a_i, b_i) \in R^{2n}_+$ where $i = 1, 2, \ldots, m$. The vector a_i is called an input vector and the vector b_i is called an output vector in the elementary process (a_i, b_i). In other words the set Z is a convex cone envelope of the finite set $\{(a_i, b_i)\}$. Thus Z can be presented in the form:

$$Z = \left\{(x, y) = \sum_{i=1}^{m} (a_i, b_i)u_i;\ u_i \geqslant 0\right\}. \tag{1}$$

If $(x, y) \in Z$, then the input vector x can be presented in the form

$$x = \sum_{i=1}^{m} a_i u_i,$$

and the output vector is in the form

$$y = \sum_{i=1}^{m} b_i u_i.$$

The number u_i is called an intensity of the i-th elementary process.

To describe an open model or just a model together with the set of elementary processes $\{(a_i, b_i)\}$ we should have a sequence of vectors $\{f(t)\}_{t=1}^{t=T}$. A trajectory in this model is a sequence $\{x(t)\}_{t=t}^{t=T}$ such that

$$(x(t), y(t)) \in Z, \qquad t = 1, \ldots, t-1 \tag{2}$$

$$x(t+1) = y(t) + f(t) \geqslant 0. \tag{3}$$

The above relations state that to get from the state vector $x(t)$ to the new state vector $x(t+1)$ at the next planning period we should first get to the transition state $y(t)$ using the set of elementary processes; and then get to the final state $x(t+1)$ by correcting $y(t)$ with the help of $f(t)$. The transition state $y(t)$ should be feasible such that $x(t+1) \geqslant 0$.

Because m, as a rule, is bigger than n there are a lot of trajectories which can start from the given initial state $x(1)$. The choice of the particular trajectory starting from the initial state $x(t)$ is done by choosing a sequence of intensity vectors $(u_1(t), u_2(t), \ldots, u_m(t))$ for $t = 1, \ldots T-1$ [6].

In the Navy sea shore rotation model which is considered in this paper we have L different locations, P different pay grades and Q maximum allowable planning periods for a person to be in any one of l locations. Thus the state of the model is characterized by a vector $x(t)$ with components $x_{lpq}(t)$ where $l = 1, \ldots, L$; $p = 1, \ldots, P$; $q = 1, \ldots, Q$; $t = 1, \ldots, t$. Here, T is the planning horizon. The value $x_{lpq}(t)$ is the number of people who at time t are at location l in pay grade p and have been at the same location for q planning periods. Thus the state of the model is a vector $x(t) = (x_{lpq}(t)) \in R_+^{L*P*Q}$ for $t = 1, \ldots, T$. By location we mean not only the geographic position of the actual duty place, but include other qualities of the duty, like whether it is at sea or on shore. So the same geographic position can have two locations: one for sea duty and one for shore duty.

To describe the elementary processes in our model let us denote e_{lpq} a unit vector in the space R^{L*P*Q*}. Then the main elementary processes are:

(1) (e_{lpq}, e_{lpq+1})—a person continues his/her duty at the same location for one more time period;
(2) (e_{lpq}, e_{l_1p1}) where $l \neq l_1$—a person changes his/her location for the next period of time;
(3) $(e_{lpq}, e_{lp+1q+1})$—a person continues his/her duty at the same location for one more time period and is promoted to the next pay grade;
(4) (e_{lpq}, e_{l_1p+11}) where $l \neq l_1$—a person changes his/her location for the next period of time and is promoted to the next pay grade.

To describe the process of leaving the Navy (controllable attrition) we will expand the space state on one dimension and denote it as $s = L * P * Q + 1$. Then $x_s(t)$ is the number of people outside of the model, i.e. $x_s(t)$ is just a big number ("general stock"). The state of the model is a vector $x(t) = (x_{lpq}(t), x_s(t)) \in R_+^{L*P*Q+1}$ for $t = 1, \ldots, T$; and the elementary process of leaving the Navy is (e_{lpq}, e_s).

The model is open, i.e. it is connected with the outside world with influx or outflux which is external to the model (uncontrollable parameters). In other words there are a sequence of vectors $f(t) \in R_+^{L*P*Q+1}$ for $t = 1, \ldots, t$. If $f_{lpq}(t) > 0$ it means that there is an influx of people to location l with pay grade p who have been at this location q planning periods. We will assume that this kind of positive influx can be only for $p = 1$, $q = 1$. That is, new people coming to location l should be in the initial pay grade and should not have been on this location before. If $f_{lpq}(t) < 0$ it means that there is an outflux of the people from the location l with pay grade p who have been on this location for q planning periods (uncontrollable attrition). We will assume that for every location l and pay grade p there is a planning or regular tour time Q_{lp} and there can be outflux (i.e. $f_{lpq}(t) < 0$) only for $q = Q_{lp}$. We made this assumption for the current version of the model because we do not have enough statistical data about attrition from different locations. It should be noted that due to our assumptions, we have $Q > \max Q_{lp}$. From equation (3) it follows that the effect of this influx will appear at the next $t + 1$ time period.

OPTIMIZATION SETTING

For the sake of simplicity and due to availability of supporting statistical data, we will assume that there is only one pay grade in the model. Therefore, the state vector is $x(t) = (x_{lq}(t), x_s(t)) \in R_+^{L*Q+1}$. Now we will present dynamic equations which describe trajectories in the model. Let I_{lq} be the initial distribution of personnel among the locations. Then for $t = 1$:

$$x_{lq}(1) = I_{lq}; \quad l = 1, \ldots, L; \quad q = 1, \ldots, Q. \tag{4}$$

Let $u_{lq}(t)$ be intensity of the first elementary process, i.e. the process of continuing a person's duty at the same location; $v_{ll_1q}(t)$ where $l \neq l_1$, is the intensity of the second elementary process, i.e. process of changing duty location; and $w_{lq}(t)$ is the intensity of the elementary process of leaving the Navy. If we accept agreement: $v_{ll_1q}(t) = u_{lq}(t)$ for $l = l_1$, and thus assume in all summations below that $v_{ll_{1q}}(t) = 0$ for $l = l_1$, then, in addition to equation (4), for $t = 1$ we will have:

$$x_{lq}(1) = u_{lq}(1) + \sum_{l_1=1}^{L} v_{ll_1q}(1) + w_{lq}(1). \tag{5}$$

The first term in this equation defines the number of people who will continue their duty at the same location; the second term defines the number of people changing their duty station and the last term defines the number of people leaving the Navy.

For $t = 2$ we will have:

$$\sum_{l_1=1}^{L} \sum_{q=1}^{Q} v_{l_1lq}(1) + f_{l1}(1) = x_{l1}(2). \tag{6}$$

That is, in the next time period at location l we will have people who are beginning their duty in this location, having moved from other locations plus the possible influx of new people from outside the model (remember $f_{l1}(t) \geqslant 0$). For $q \geqslant 2$ and $q \leqslant Q$ we will have:

$$u_{lq-1}(1) + f_{lq}(1) = x_{lq}(2). \tag{7}$$

That is, people in location l who have been at the location q periods are people who were at this location before minus possible outflux from the model if $q = Q_l$.

Due to our assumption about influx/outflux vectors $f_{lq}(t) = 0$ for $t = 1, \ldots, T-1$; $l = 1, \ldots, L$; $q = 1, \ldots, Q$; $q \neq 1$; $q \neq Q_l$, instead of (7) we will have

$$u_{lq-1}(1) = x_{lq}(2). \tag{8}$$

Equations (5) and (6) describe all possible one period trajectories which begin from the state vector $x(1)$ with components $x_{lq}(1)$ and finish in the state vector $x(2)$ with components $x_{lq}(2)$. In the next planning period $t = 2$, the output state vector $x(2)$ will be the input state vector for the next period of a trajectory; and, therefore, like equation (5) we will have the equation which defines intensitities for $t = 2$:

$$x_{lq}(2) = u_{lq}(2) + \sum_{l_1=1}^{L} v_{ll_1q}(2) + w_{lq}(2). \tag{9}$$

For arbitrary $t \leqslant T$ from equations (5), (6), (7) and (9) we will have for $q = 1$:

$$\sum_{l_1=1}^{L} \sum_{q=1}^{Q} v_{l_1lq}(t) + f_{l1}(t) = x_{l1}(t+1) = u_{l1}(t+1) + \sum_{l_1=1}^{L} v_{ll_11}(t+1) + w_{l1}(t+1), \tag{10}$$

and for $q \neq 1$

$$u_{lq-1}(t) + f_{lq}(t) = x_{lq}(t+1) = u_{lq}(t+1) + \sum_{l_1=1}^{L} v_{ll_1q}(t+1) + w_{lq}(t+1). \tag{11}$$

Equations (10) and (11) together with equation:

$$I_{lq} = x_{lq}(1) = u_{lq}(1) + \sum_{l_1=1}^{L} v_{ll_1q}(1) + w_{lq}(1), \tag{12}$$

define the sequence of intensities for the given trajectory $\{x(t)\}_{t=t}^{t=t}$.

To define the set of feasible trajectories we will introduce values D_l which are minimum manning requirements at locations $l = 1, \ldots, L$. Then inequality

$$\sum_{q=1}^{Q} x_{lq}(t) \geqslant D_l \tag{13}$$

will describe the minimum manning requirement for any location and any period of time $1 \leqslant t \leqslant T$. Also let F_1 be a set of forbidden changes of locations, i.e. F_1 is a set of pairs (l, l_1) such that

$$v_{ll_1q}(t) = 0; \quad (l, l_1) \in F_1. \tag{14}$$

An example of a forbidden change of location is a change from sea duty again to sea duty. Then relations (11)–(14) will define the set of all feasible trajectories $\{x(t)\}_{t=t}^{t=T}$ starting from the same initial state $x(1)$ which is defined by the initial distribution as described in equation (4). To describe the optimization setting of the model we should introduce different costs.

Let $c^1_{ll_1}$ be the cost to relocate a person from location l to location l_1. The cost of keeping a person at the same location inside of regular tour time is c^2_1, and penalty for keeping a person outside of the location's regular tour time Q_l is p^2_l. Some location changes can be rather undesirable. For example, an undesirable change of location may be a change from sea duty in Hawaii to shore duty in Hawaii. Let F_2 be the set of all undesirable changes (l, l_1), and $p^1_{ll_1}$ be the penalty for relocating a person using an undesirable location change, i.e. $(l, l_1) \in f_2$.

For each location there are minimum manning constraints (13) and manning goals G_l. If the minimal manning constraints are not met the trajectory will be infeasible. We refer to these kind of constraints as "hard constraints". Quite the opposite, if manning (requisition) goals G_l are not met the trajectory will remain feasible and there will be only penalties g_l for not keeping these manning goals G_l in location l. Thus, goal constraints can be called "mild constraints".

Finally, let P be the penalty for letting a person leave the Navy. Then the problem of choosing an optimal trajectory consists of minimizing the objective function:

$$\sum_{t=1}^{T-1} \sum_{l=1}^{L} \left(\sum_{q=1}^{Q} \left(\sum_{l_1=1}^{L} c^1_{ll_1} * v_{ll_1q}(t) + \sum_{(l,l_1)\in F_2} p^1_{ll_1} * v_{ll_1q}(t) \right) \right.$$
$$\left. + \sum_{q=1}^{Q_l} c^2_l * u_{lq}(t) + \sum_{q=Q_l+1}^{Q} p^2_l * u_{lq}(t) + g_l * \left| \sum_{q=1}^{Q} x_{lq}(t) - G_l \right| + P * \sum_{q=1}^{Q} w_{lq}(t) \right) \tag{15}$$

under constraints (11)–(14). The first summation in equation (15) is the cost of all location changes, the second is all penalties for using undesirable location changes, the third is the cost of keeping a person in a location inside a regular tour time, the fourth is all penalties for keeping a person in a location beyond the regular tour time, the fifth is all penalties for not keeping manning goals for a location, and the last summation is penalties for letting people leave the Navy.

In this optimization problem variables are intensities: $u_{lq}(t)$, $v_{ll_1}q(t)$, $w_{lq}(t)$ where $l, l_1 = 1, \ldots, l$; $q = 1, \ldots, Q$; $t = 1, \ldots, T - 1$.

NETWORK REPRESENTATION OF THE MODEL

The described model can be presented as a network with side constraints. To describe the "network" part of the model we will define a node for every triple (l, q, t) where $l = 1, \ldots, L$; $q = 1, \ldots, Q$; $t = 1, \ldots, t$. Thus a location l at the time t will be characterized by Q nodes. Together with $L * Q * T$ nodes we define a node which we call the general stock to interpret the outside world. Equations (10)–(12) describe arc connections between nodes plus possible demand and supply at every node. For example, equation (12) means that every node corresponding to triple $(l, q, 1)$ has supply I_{lq}. That coincides with the meaning of the value I_{lq} for the given l where $q = 1, \ldots, Q$ for the initial state at location l. The right-hand side of (12) gives arcs connecting this node with other nodes of the model. The term $u_{lq}(1)$ represents the arc which connects the given node $(l, q, 1)$ with node $(l, q + 1, 2)$, if $q < Q$, which agrees with the definition of intensity $u_{lq}(1)$ as the intensity of the process of staying at the same location one more period of time. The same node $(l, q, 1)$ is connected with nodes $(l_1, 1, 2)$ where $l_1 \neq l$ which corresponds to the variable $v_{ll_1q}(1)$. Of course, if $(l, l_1) \in F_1$ the above arc would be absent because the corresponding location change is forbidden by the definition of F_1. Recall that due to equation (14), if $(l, l_1) \in F_1$, there is no arc connecting node $(l, q, 1)$ with the node $(l_1, 1, 1)$ since we are not using forbidden changes of loctions. Finally, the term $w_{lq}(1)$ represents the arc connecting the given node $(l, q, 1)$ with the general stock. We will assume that $w_{l1}(t) = 0$ for all $l = 1, \ldots, L$; $t = 1, \ldots, T - 1$; i.e. people who just began their service at location l cannot leave the Navy.

The same type of arcs will connect the next layer of nodes $(l, q, 2)$, where $l = 1, \ldots, L$; $q = 1, \ldots, Q$ with the next time-layer $(l, q, 3)$, where $l = 1, \ldots, L$; $q = 1, \ldots, Q$, or with the general stock node. This network construction will continue for $t < T$. The last layer of nodes (l, q, T) will have only arcs connecting these nodes with the general stock node. This means that these nodes represent the final stage of the model. The capacity of every arc is infinite. Unit costs for the arcs are defined by equation (15). Particularly, for the arc connecting the node (l, q, t) with the node $(l, 1, t + 1)$ which corresponds to the variable $v_{ll_1q}(t)$, where $l \neq l_1$, the cost is equal to $c^1_{ll_1}$, or, in the case of $(l, l_1) \in F_2$, the unit cost for the same arc is equal to $(c^1_{ll_1} + p^1_{ll_1})$. For the arc connecting the node (l, q, t) with the node $(l, q + 1, t + 1)$ the cost is equal to c^2_l, if $q \leqslant Q_1$, or the cost is equal to p^2_l, if $q > Q_l$. For the arc connecting the node (l, q, t) with the general stock node the unit cost is equal to P. Finally, arcs connecting the last time-layer of nodes (l, q, T) with the general stock node have zero unit costs.

The node $(l, 1, t)$ for $l = 1, \ldots, L$; $1 < t < T$ will have supply $f_{l1}(t) \geqslant 0$ which corresponding to the inflow of new people to location l from the outside world. In the same mode, the node $(l, Q_l + 1, t)$ for $l = 1, \ldots, L$; $1 < t < T$ will have demand $f_{lQ_l+1}(t) \leqslant 0$ which corresponds to the outflow of people from location l after regular tour time for this location. All other nodes, except the initial and last time-layer nodes, have zero supply and demand. As we already mentioned at the initial time-layer any node $(l, q, 1)$ has supply equal I_{lq}. For the final stage node we want to produce an "ideal" manning for a location. For this reason, for a node (l, q, T) we will define the demand as G_l/Q_l, if $q \leqslant Q_1$, and zero, if $q > Q_l$.

Incidentally, due to the well known "boundary effect", in all our runs we will consider only results for $t < T$, where T is the planning horizon.

In Fig. 1 we depict a network for the model for two locations $L = 2$ and time horizon $T = 5$. The first location has a regular tour time of $Q_1 = 3$, and the second location has a regular tour time of $Q_2 = 2$. In the presented case there are no forbidden arcs. In Fig. 1 every node is marked by the triple (l, q, t), where $l = 1, 2$ is a location number, $q = 1, 2, 3$ is tour time at the location, and $t = 1, 2, 3, 4, 5$ is planning time. We did not depict the last row of nodes for the second location (corresponding to $q = 3$), even though these nodes, generally speaking, can be used.

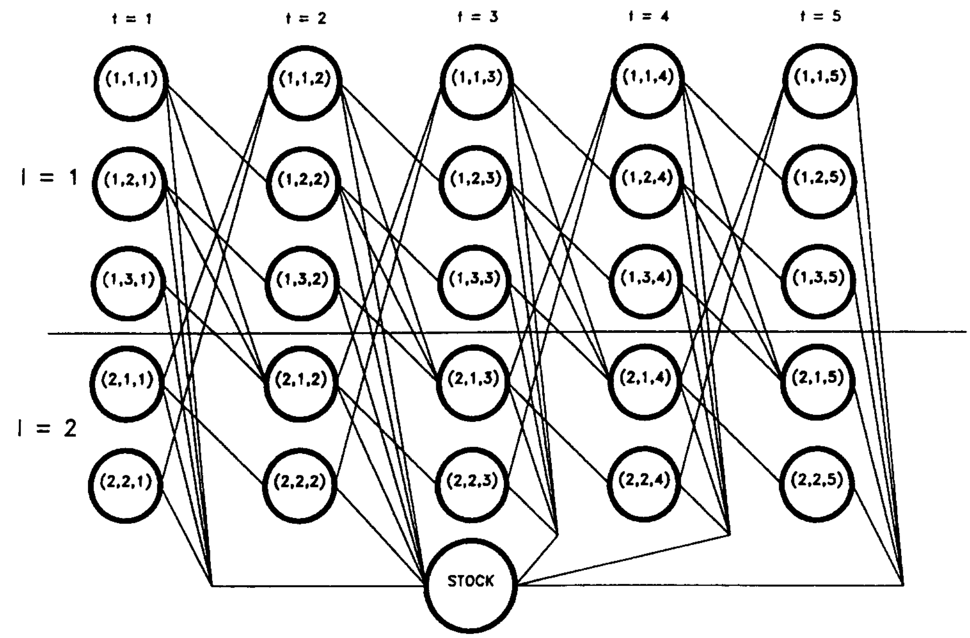

Fig. 1. Network of the model for the case $L = 2$; $T = 5$.

Thus, like in [2] the network part of the model is a dynamic network which is growing toward time horizon T. The differences are in the side constraints in equation (13) and summation goals in equation (15). To present the summation goal as a side constraint we will introduce new non negative variables $z_l^1(t)$, $z_l^2(t)$ where $l = 1, \dots, L$; $t = 1, \dots, T$ which are defined as:

$$z_l^1(t) - z_l^2(t) = G_l - \sum_{q=1}^{Q} x_{lq}(t) = G_l - \sum_{q=1}^{Q}\left(u_{lq}(t) + \sum_{l_1=1}^{L} v_{ll_1q}(t) + w_{lq}(t)\right). \tag{16}$$

The last equality is taken from equation (11). With the help of this definition the objective function (15) can be rewritten as:

$$\sum_{t=1}^{T-1}\sum_{l=1}^{L}\left(\sum_{q=1}^{Q}\left(\sum_{l_1=1}^{L} c_{ll_1}^1 * v_{ll_1q}(t) + \sum_{(l,l_1)\in F_2} p_{ll_1}^1 * v_{ll_1q}(t)\right)\right.$$
$$\left. + \sum_{q=1}^{Q_l} c_l^2 * u_{lq}(t) + \sum_{q=Q_l+1}^{Q} p_l^2 * u_{lq}(t) + g_l * (z_l^1(t) + z_l^2(t)) + P * \sum_{q=1}^{Q} w_{lq}(t)\right). \tag{17}$$

Equation (16) presents additional side constraints to our network model.

Before describing the practical runs of the model we want to mention that for computer implementation we are using Kennington's code for networks with side constraints [7]. This code produces an optimal solution for the given problem but it does not look for the optimal nteger solution. It is well known, that generally speaking the described problem, i.e. a network problem with side constraints, is a N.P.-hard problem. So attempting to solve this problem using a general integer LP code for the sizes which we had for practical runs [approx. 7000–8000 variables, 2000–3000 relations including network type equations (9)–(12)] would have been absolutely dreadful. For this reason we decided to use the Kennington code which rather effectively can find an optimal solution for the given problem and then we applied a rounding routine to get the approximate integer solution. Our rounding routine begins rounding from $t = 1$ where, due to the initial setting, all supply values are integers. Then it follows from one time-layer to another finding for every arc the lowest cost integer flow so that, at every node, flow balance will be conserved and constraints (13) will not be broken. By flow balance, we mean preserving the equality of influx flow plus supply to outflux flow plus demand for every node. Because a linear function like equation (17) is continuous we know that if the integer approximation or heuristic solution is close to the optimal non integer solution, we will not lose much from the point of view of the objective function. In all our practice runs our rounding solution differs by an absolute value from the optimal non integer solution produced by the Kennington program by not more than one for any arc flow.

MODEL SETUP FOR PRACTICAL RUNS

Historical sea shore distribution and inventory data were obtained from the Enlisted Master Record (EMR) for the Aviation Antisubmarine Warfare Technician (AX) rating. The data were aggregated so that groups are defined by geographic location and composite. The composite is a type of duty classification of the billets (jobs) based on whether the billets are in the continental U.S. (CONUS) or overseas [outside the continental U.S. (OUTUS)] and whether they are designated as shore duty or sea duty. There are four composites—CONUS shore, CONUS sea, OUTUS shore and OUTUS sea. The aggregation of data resulted in the following $L = 15$ locations:

Geographic location	Composite
1. CONUS	CONUS Shore
2. CONUS	CONUS Sea
3. Alaska	OUTUS Shore
4. Alaska	OUTUS Sea
5. Bermuda	OUTUS Shore
6. Diego Garcia	OUTUS Shore
7. Diego Garcia	OUTUS Sea
8. Hawaii and Guam	OUTUS Shore
9. Hawaii and Guam	OUTUS Sea
10. Iceland and The Netherlands	OUTUS Shore
11. Iceland and The Netherlands	OUTUS Sea
12. Japan and The Philippines	OUTUS Shore
13. Japan and The Philippines	OUTUS Sea
14. Spain and Sicily	OUTUS Shore
15. Spain and Sicily	OUTUS Sea

These locations were chosen because an approximate estimate of the cost of moving a person to the other locations [coefficient $c^1_{ll_1}$ in equation (17)] and tour length $(Q_l, l = 1, \ldots, L)$ could reasonably be assigned for each location. Maximum allowable planning periods for a person to be in any one of $L = 15$ locations $Q > \max_{l=1,L} Q_l$ was chosen as $Q = 5$. In Tables 1 and 2 the regular tour lengths and cost of moving coefficients are presented respectively.

Within each location, the inventory was divided by tour time. The initial distribution of inventory by location and tour time $(I_{lq}; l = 1, \ldots, L; q = 1, \ldots, Q)$ is given in the Table 3.

The distribution of authorized billets for each location was used to determine the manning goals for manning for each location (G_l). (See Table 4.)

Gain and loss flows [i.e. influx and outflux to the model from outside world—$f_{lq}(t)$ in equations (10) and (11)] were determined by assuming a steady state at a flow rate of 31%. The 31% rate was applied to the beginning inventory for each location to determine the influx and outflux values. Therefore, values for $f_{lq}(t)$ do not depend on t, and $f_{l1} = -f_{lQ_l}$; $l = 1, \ldots, 15$. The flow data are shown in Table 5.

Thus, data for each location included: tour length (Table 1); moving costs (Table 2); initial inventory (Table 3); manning goals (Table 4); number of people gained or lost per period of time (Table 5).

Two types of forbidden changes of locations (the set F_1) were defined. The first is moving from one composite to the same composite and the second is moving from CONUS sea $(l = 2)$ to a location in an overseas composite $(l > 2)$. Also, two types of undesirable changes of location (the set F_2) are defined. The first is assigning people to the same geographic location, but different composite and the second is moving people to an overseas location from an overseas location.

Table 1. Regular tour length by location

Location #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Regular Tour Length	3	4	2	2	3	1	1	3	3	3	2	3	3	3	2

Table 2. Cost of moving a person from one location to another

Location	1	2	3	4	5	6	7	8	9
1	0	2866	5833	5833	7044	6658	6658	5325	5325
2	2866	0	5833	5833	7044	6658	6658	5325	5325
3	5833	5833	0	0	12498	8001	8001	11784	11784
4	5833	5833	0	0	12498	8001	8001	11784	11784
5	7044	7044	12498	12498	0	8869	8869	7893	7893
6	6658	6658	8001	8001	8869	0	0	7177	7177
7	6658	6658	8001	8001	8869	0	0	7177	7177
8	5325	5325	11784	11784	7893	7177	7177	0	0
9	5325	5325	11784	11784	7893	7177	7177	0	0
10	8325	8325	13983	13983	11979	10486	10486	13258	13258
11	8325	8325	13983	13983	11979	10486	10486	13258	13258
12	4971	4971	12721	12721	9773	7036	7036	9557	9557
13	4971	4971	12721	12721	9773	7036	7036	9557	9557
14	5304	5304	13685	13685	7808	7488	7488	9655	9655
15	5304	5304	13685	13685	7808	7488	7488	9655	9655

Location	10	11	12	13	14	15
1	8325	8325	4971	4971	5304	5304
2	8325	8325	4971	4971	5304	5304
3	13983	13983	12721	12721	13685	13685
4	13983	13983	12721	12721	13685	13685
5	11979	11979	9773	9773	7808	7808
6	10486	10486	7036	7036	7488	7488
7	10486	10486	7036	7036	7488	7488
8	13258	13258	9557	9557	9655	9655
9	13258	13258	9557	9557	9655	9655
10	0	0	12314	12314	8166	8166
11	0	0	12314	12314	8166	8166
12	12314	12314	0	0	11477	11477
13	12314	12314	0	0	11477	11477
14	5304	5304	11477	11477	0	0
15	5304	5304	11477	11477	0	0

Table 3. Initial inventory per tour time per location

	Tour Time				
Location	1	2	3	4	5
1	384	337	220	81	19
2	337	264	183	70	9
3	7	3	2	0	0
4	0	0	0	0	0
5	1	2	6	1	0
6	12	4	0	0	0
7	0	0	0	0	0
8	11	10	13	7	0
9	40	47	25	8	6
10	2	5	4	0	0
11	0	0	0	0	0
12	13	12	10	4	1
13	8	5	2	0	0
14	13	17	7	3	0
15	0	0	0	0	0

Table 4. Manning requirements goals per location

Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Manning Goals	995	652	7	6	17	13	5	32	107	5	5	31	34	35	10

Table 5. Gain/loss flows by location

Location	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
In / Out Flows	321	266	2	2	3	0	0	13	39	1	2	12	5	10	3

Table 6. Forbidden location changes

Location from	Location to
2	3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15
3	5, 6, 8, 10, 12, 14
4	7, 9, 11, 13, 15
5	3, 6, 8, 10, 12, 14
6	3, 5, 8, 10, 12, 14
7	4, 9, 11, 13, 15
8	3, 5, 6, 10, 12, 14
9	4, 7, 11, 13, 15
10	3, 5, 6, 8, 12, 14
11	4, 7, 9, 13, 15
12	3, 5, 6, 8, 10, 14
13	4, 7, 9, 11, 15
14	3, 5, 6, 8, 10, 12
15	4, 7, 9, 11, 15

Tables 6 and 7 show the forbidden and undesirable location changes respectively. Particularly, all no cost moves are undesirable. (See Table 2.)

To differentiate the importance of reaching manning goals in different locations, coefficient g_l in equation (17) was defined as 1 for CONUS shore; 1000 for any sea location and 250 for other locations. The four policy object-goals for which the priority could be changed were: minimizing moving costs; meeting manning requirements; keeping a person for the regular tour time; not using undesirable changes of location. These four policies correspond to the objective function (17) coefficients $c^1_{ll_1}$; g_l; p^2_l; $p^1_{ll_1}$; respectively. Values for the coefficient c^2_l were defined as 1, and the value of P was taken as a "big number", i.e. a number which exceeds all other coefficients together with weights of at least three decimal orders. This choice for P corresponds to the assumption that controlling attrition by location is a very undesirable process. Priority between the four object-goals was done with the help of multiplicative weight coefficients in the same manner as in the Enlisted Personnel Allocation and Nomination System—EPANS [4].

RESULTS OF PRACTICAL RUNS

The AX rating is unusual in that it had more inventory than authorized billets during the data collection period. That means that the manning levels are often over 100% for the locations, which affects the object of meeting manning requirement goals. This effect is especially strong for location 2 (CONUS sea) where the manning goal is too low for the initial inventory or periodical influx/outflux of people to/from the location. Notice, from Table 5 that the constant outflux from location 2 is 266 (remember, that in the model discussed we assume that influx equals outflux for every location). Also, as is shown in Table 1 the regular tour length for this location is 4. This means that we have to have $266 * 4 = 1064$ persons in this location if we run the model with a planning horizon of more than 4 years. This number, 1064, considerably exceeds the manning goal 652 for location 2 (see Table 4).

In the first runs of the model we wanted to compare the effects of trading off between the different object-goals. For this reason, we fixed the planning horizon at 7 yr ($T = 7$) and used the data set

Table 7. Undesirable location changes

Location from	Location to
3	4, 7, 9, 11, 13, 15
4	3, 5, 6, 8, 10, 12, 14
5	4, 7, 9, 11, 13, 15
6	4, 7, 9, 11, 13, 15
7	3, 5, 6, 8, 10, 12, 14
8	4, 7, 9, 11, 13, 15
9	3, 5, 6, 8, 10, 12, 14
10	4, 7, 9, 11, 13, 15
11	3, 5, 6, 8, 10, 12, 14
12	4, 7, 9, 11, 13, 15
13	3, 5, 6, 8, 10, 12, 14
14	4, 7, 9, 11, 13, 15
15	3, 5, 6, 8, 10, 12, 14

Table 8. Trade off between different goals

First Goal in Policy Order	Policy Object-Goals: Average Moving Cost	Meeting Manning Requirements Compliance	Keeping within Tour Time Compliance	Number of Undesirable Location Changes
Minimize Moving Cost	$1,692	69%	92%	244
Meeting Manning Requirements	$4,162	92%	94%	0
Keeping within Tour Time	$6,006	70%	97%	248
Minimize Undesirable Location Changes	$3,959	78%	83%	0

which characterized the model (see previous paragraph) changing only the order of object-goals for different runs. Because of the preemptive type approach to the multiobjective problem of optimization, results of the runs were more responsive to the first object-goal in the object-goal hierarchy for different runs. These results are summarized in Table 8. The percentage of compliance with the manning requirements object-goal for the locations was measured as:

$$R_l(t) = \left(1 - \left|\sum_{q=1}^{Q} x_{lq}(t) - G_l\right| \Big/ G_l\right) * 100\%. \tag{18}$$

That is, if the manning requirements goal for location l at time t is met, i.e.

$$\sum_{q=1}^{Q} x_{lq}(t) = G_l,$$

then $R_l(t) = 100\%$, i.e. compliance with manning requirements in this case is 100%. Also any deviation from the goal below or above the goal manning decreases the above compliance equally. In Fig. 2 we depict the graph of the compliance function for the case of location 2.

To measure model performance of keeping manning requirements goals we also used the usual percentage:

$$r_l^1(t) = \left|\sum_{q=1}^{Q} x_{lq}(t) - G_l\right| \Big/ G_l * 100\%. \tag{19}$$

In Table 9 we demonstrate the dynamics of both types of measurements for meeting manning requirements object-goals. As we mentioned above, in all goal performance measuring we never use the last time period of planning horizon to avoid the "boundary effect". For example in Tables 8 and 9 where the planning horizon is $T = 7$ we use only the first six periods for calculations. Moreover, in the case of meeting manning requirements in calculations of average percentage/compliance we did not use the first period. This is because, in the first period, manning of

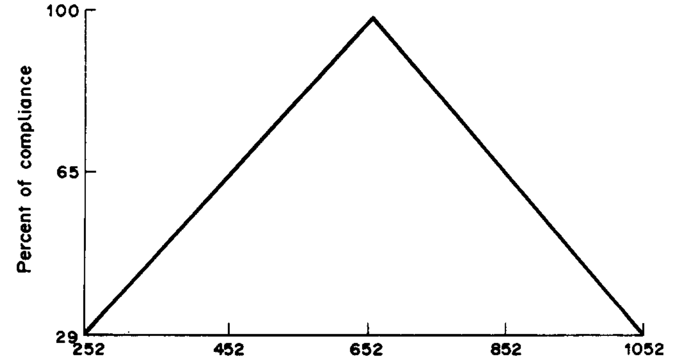

Fig. 2. Compliance graph for the location $l = 2$.

Table 9. Dynamics of percentage of meeting/compliance manning requirements goal. Time horizon is 7. Averaging is done by last 5 periods

	Time Periods						
Location	1	2	3	4	5	6	Average
1	104%	122%	120%	139%	154%	169%	141%
	95%	77%	80%	61%	46%	31%	59%
2	132%	153%	171%	142%	121%	100%	137%
	68%	47%	29%	58%	79%	100%	62%
3	171%	100%	100%	100%	100%	100%	100%
	29%	100%	100%	100%	100%	100%	100%
4	0%	100%	100%	100%	100%	100%	100%
	0%	100%	100%	100%	100%	100%	100%
5	59%	100%	100%	100%	100%	100%	100%
	59%	100%	100%	100%	100%	100%	100%
6	123%	100%	100%	100%	100%	100%	100%
	77%	100%	100%	100%	100%	100%	100%
7	0%	100%	100%	100%	100%	100%	100%
	0%	100%	100%	100%	100%	100%	100%
8	128%	106%	115%	122%	109%	100%	110%
	72%	94%	84%	78%	91%	100%	89%
9	118%	109%	109%	109%	105%	100%	108%
	82%	91%	91%	91%	95%	100%	93%
10	220%	100%	100%	100%	100%	100%	100%
	19%	100%	100%	100%	100%	100%	100%
11	0%	100%	100%	100%	100%	100%	100%
	0%	100%	100%	100%	100%	100%	100%
12	129%	116%	116%	116%	106%	100%	109%
	71%	84%	84%	84%	94%	100%	89%
13	44%	100%	100%	100%	100%	100%	100%
	44%	100%	100%	100%	100%	100%	100%
14	114%	100%	100%	100%	100%	100%	100%
	86%	100%	100%	100%	100%	100%	100%
15	0%	100%	100%	100%	100%	100%	100%
	0%	100%	100%	100%	100%	100%	100%
Average	89%	107%	109%	108%	106%	105%	107%
	43%	92%	91%	91%	93%	95%	92%

locations coincides with initial manning and only for $t > 1$ can the optimization mechanism of the model really play its role in changing the manning. To get Table 8 we fixed the sequence of the object-goals in the pattern presented in the talbe; then we changed this sequence cyclicly such that for every run a different object-goal was first in the policy hierarchy and all object-goals were first at least once.

To better understand the trade off between the goal of minimizing moving costs and meeting manning requiremrnts we ran the model with two different orders of object-goals. The first order was: minimizing moving costs; meeting manning requirements; keeping a person for the regular tour time; Not using undesirable changes of location. The second order was; meeting manning requirements; minimizing moving costs; keeping a person for the regular tour time; not using undesirable changes of location. In the first case, the model required only 516 location changes with an average cost of one move of \$1692, but compliance with manning requirements was only 69%. In the second case, the model made 1144 location changes with an average cost of \$4142, but compliance with manning requirements was 92%. Thus, to gain an additional 20% in meeting manning requirements compliance even the optimization model spent nearly two and one-half times more money for location changes, as well as making over two times more location changes.

As we mentioned at the beginning of this section in the considered rating AX, we have more persons in the rating than it requires to meet manning requirements object-goals. This is an especially noticeable phenomena for location 2. When the run time approaches the time horizon T the optimization process tries to move excess people from locations with big goal costs g_l to locations with lesser goal costs. This happens because the final stage for every location is the "golden state", i.e. demand at the final stage $t = T$ provides $< 100\%$ of manning requirements for every location. The dynamics of meeting manning requirements per location for time horizon $T = 7$ is shown in Table 9. As already noted, calculating the average manning goal compliance is

Table 10. Dynamics of manning of locations

	Time Periods					
Location	1	2	3	4	5	6
1	1041	1220	1195	1386	1536	1682
2	863	997	1113	926	788	652
3	12	7	7	7	7	7
4	0	6	6	6	6	6
5	10	17	17	17	17	17
6	16	13	13	13	13	13
7	0	5	5	5	5	5
8	41	34	37	39	35	32
9	126	117	117	117	112	107
10	11	5	5	5	5	5
11	0	5	5	5	5	5
12	40	36	36	36	33	31
13	15	34	34	34	34	34
14	40	35	35	35	35	35
15	0	10	10	10	10	10

done without taking into account the initial inventory distribution, i.e. for $t \neq 1$, because model optimization is in effect only for $t > 1$.

Data for Table 9 were collected from the second case of object-goals ordering, discussed above, where the moving costs were traded off against meeting manning goals. Except for the initial data distribution, meeting manning requirements compliance of 100% means that we have more people at a location than necessary [see equation (18) for compliance calculation]. The dynamics of location manning corresponding to Table 9 are shown in Table 10.

Calculation of the optimal solution for time horizon $T = 7$ takes approx. 3 min of CPU time of the IBM 4341/12 using FORTRAN as the programming language. We also ran the model for the same sequence of objectives for 9 and 11 time horizons which takes about 7 and 11 min, respectively. Tables 11 and 12 show goal compliance numbers for these time horizons. As we can see, the pattern of meeting manning requirements remains close to the same with the different time horizons.

CONCLUSION

One of the important manpower planning problems in the Navy is predicting the effect of different policies on rotation of personnel to different locations. For example, how do you compare the impact of decreasing the budget for location changes (so called Permanent Change of Station—PCS cost) on the percentage of meeting manning requirements for different locations? Or, vice versa, what is the impact of decreasing the amount of people for some locations or decreasing their tour times on increasing the moving cost budget? The model developed in this paper will help to answer many of these "what if" questions. It has enough flexibility to

Table 11. Dynamics of percentage of meeting manning requirements goal. Time horizon is 11. Averaging is done by last 10 periods

	Time Periods										
Locat.	1	2	3	4	5	6	7	8	9	10	Aver.
1	105%	117%	109%	115%	112%	109%	125%	138%	154%	167%	127%
2	132%	161%	187%	178%	183%	187%	163%	143%	121%	103%	158%
3	29%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
4	0%	100%	116%	100%	100%	100%	100%	100%	100%	100%	102%
5	58%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
6	123%	100%	100%	100%	100%	100%	100%	100%	100%	108%	101%
7	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
8	128%	106%	116%	122%	122%	122%	122%	122%	122%	100%	117%
9	118%	110%	110%	110%	110%	110%	110%	110%	105%	100%	108%
10	220%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
11	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
12	129%	116%	116%	116%	116%	116%	116%	116%	106%	100%	86%
13	44%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
14	114%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
15	0%	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
Aver.	89%	107%	109%	110%	109%	110%	109%	109%	107%	105%	108%

Table 12. Dynamics of percentage of meeting manning requirements goal. Time horizon is 9. Averaging is done by last 8 periods

	Time Periods								
Location	1	2	3	4	5	6	7	8	Average
1	105%	124%	125%	118%	125%	139%	154%	169%	136%
2	132%	150%	163%	175%	163%	142%	120%	100%	144%
3	171%	100%	100%	100%	100%	100%	100%	100%	100%
4	0%	100%	100%	100%	100%	100%	100%	100%	100%
5	58%	100%	100%	100%	100%	100%	100%	100%	100%
6	123%	100%	100%	100%	100%	100%	100%	100%	100%
7	0%	100%	100%	100%	100%	100%	100%	100%	100%
8	128%	106%	116%	122%	122%	122%	109%	100%	114%
9	118%	109%	109%	109%	109%	109%	105%	100%	92%
10	220%	100%	100%	100%	100%	100%	100%	100%	100%
11	0%	100%	100%	100%	100%	100%	100%	100%	100%
12	119%	116%	116%	116%	116%	116%	106%	100%	113%
13	44%	100%	100%	100%	100%	100%	100%	100%	100%
14	114%	100%	100%	100%	100%	100%	100%	100%	100%
15	0%	100%	100%	100%	100%	100%	100%	100%	100%
Average	89%	107%	109%	109%	109%	109%	106%	105%	108%

accommodate the different parameters needed to make reliable predictions, and it does not require excessive computer time. The computer implementation of the model is done in a such a way that the main model parameters are easy to change, which gives it the capability to answer many "what if" questions. The model produces reasonable results, and sensitivity analysis can be performed on some of the parameters to estimate impacts on PCS cost or manning requirements. For example, what would be the impact of changing the regular influx/outflux from a location? This influx/outflux depends not only on the internal Navy policies, but also on other external factors, such as the unemployment rate.

Due to the built in flexibility to accommodate different parameters, the reasonable computer time required for each run, and the dynamic structure, the model has the potential to be a valuable sea/shore rotation planning tool for the Navy.

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